Chapter 9: Discussion and Conclusions

In this thesis, it has been demonstrated that the well-known Bohmian model for quantum mechanics can be made compatible with the laws of conservation of energy and momentum. This has been achieved by constructing a Lagrangian formulation of the model, so that the required conservation is then assured by Noether's theorem. Although this conservation is then known to be present in general terms, extracting a detailed description of it in terms of energy-momentum tensors was found to be not at all straightforward. First, it was necessary to realize that the usual energy-momentum tensors $T^{\mu\nu}{}_{\text{field}}$ and $T^{\mu\nu}{}_{\text{particle}}$ that appear in such a formulation had, in this case, to be augmented by a third tensor $T^{\mu\nu}{}_{\text{interaction}}$. Furthermore, attempting to obtain a specific expression for $T^{\mu\nu}{}_{\text{interaction}}$ by re-deriving Noether's proof from first principles was found to lead to difficulties and ambiguities in the non-relativistic case.

The chosen way forward was to formulate the details of a **relativistic** Lagrangian model and then take the non-relativistic limit to obtain the appropriate formalism corresponding to Bohm's model. Although the relativistic case proved to be straightforward, the taking of the non-relativistic limit was also found to involve subtleties and to require care. During this procedure, it was necessary to scrutinize the physical interpretation of the various expressions that arose and to clarify the physical meaning of symmetric and non-symmetric energy-momentum tensors. The resulting formalism was then found to have the desired properties for demonstrating conservation.

The construction of a Lagrangian formulation of Bohm's model necessarily leads to some modification of the relevant field equation, i.e., of the Schrodinger equation. This has resulted in perhaps the most intriguing aspect of the present approach, namely that the modified Schrodinger equation is found to be automatically of a special form that reduces back to the standard Schrodinger equation[1] in the usual, non-relativistic case of no creation or annihilation of particles, thereby maintaining compatibility with all the relevant experimental evidence.

Concerning this restriction to conserved particle number in the non-relativistic case, it may be observed that the divergences of the various parts of the energy-momentum tensor become somewhat trivial under this assumption. However, this does not affect the basic result that energy and momentum conservation have been successfully introduced into Bohm's model. Of course, the fact also remains that Bohmian mechanics is not a widely accepted interpretation of quantum mechanics. Nevertheless, the aim here has simply been to answer the question of whether this model can be compatible with the usual conservation laws (this question having been published in several places), not to argue for the superiority of the model in other ways.

Finally it should also be noted that, from a metaphysical point of view, the present work establishes that the laws of energy and momentum conservation are quite compatible with attempts to formulate an interpretation of quantum mechanics incorporating realism.

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[1] once the Bohmian constraint $\mathbf{v} = \nabla S/m$ is applied.