Chapter 1: Introduction

This thesis looks at a particular interpretation of the formalism of quantum mechanics, viz., the model proposed by David Bohm. The aim is not to argue for or against this model, since the whole interpretation question for quantum mechanics is an area of much controversy. Rather, the aim is to resolve a precisely defined physical and mathematical problem that has been highlighted by several authors as being a possible deficiency of the model. It is demonstrated here that this feature of Bohm's model, namely that it does not conserve energy and momentum, can be successfully eliminated if desired.

Advocates of Bohm's model can, of course, claim that it is already both empirically adequate and logically consistent without introducing such conservation. Nevertheless, there seems to be a general view, shared by supporters of the model, that the possibility of restoring energy and momentum conservation remains an interesting and aesthetically appealing idea.

The structure of the thesis is as follows:

Chapter 2 provides a general discussion of the development of quantum mechanics and the problem of its interpretation. It considers the Copenhagen interpretation, the Measurement Problem and the possibility of hidden variables.

Chapter 3 summarizes the basic structure of Bohm's model for quantum mechanics. It describes the model's derivation from the equation of continuity and compares the modern minimalist version of the model with Bohm's original version. Expressions for Bohm's "quantum potential" are derived in preparation for later use in the thesis. The fact that Bohm's model does not conserve energy and momentum is then highlighted, this aspect of the model being the main focus of subsequent chapters. Finally, possible extensions to Bohm’s model that have been suggested by other authors are discussed.

In chapter 4, the Lagrangian formalism is outlined in preparation for applying it to Bohm’s model. The eventual aim is to introduce energy and momentum conservation via Noether’s theorem. Examples of a Lagrangian for particle motion and Lagrangian densities for free field evolution are first discussed, followed by sample Lagrangian densities for a particle and field in interaction. These expressions serve as possible analogies and guides towards a Lagrangian density for Bohm’s model. Finally, an earlier attempt at a Lagrangian formalism for Bohm’s model, proposed by Squires, is summarized and discussed.

In chapter 5, a Lagrangian density suitable for Bohm’s model is introduced. It is then demonstrated that this expression yields the usual equation of motion for the Bohmian particle. Such a Lagrangian formulation characterizes Bohm’s model as an interacting particle-field system and pursuing this approach necessarily causes some modification to the Schrodinger equation. It is shown, however, that the particular modification introduced by the Lagrangian density proposed here does not compromise the Schrodinger equation’s standard, experimentally-verified predictions.

Chapter 6 summarizes the general theory of energy and momentum conservation for particle-field systems in terms of the divergence of energy-momentum tensors. It then tentatively considers the application of this formalism to Bohm’s model and highlights some difficulties that arise.

Chapter 7 proceeds to resolve these difficulties encountered in the non-relativistic theory by instead formulating a relativistic treatment, using a Klein-Gordon version of Bohm's model published by de Broglie. The mathematical proof of Noether's theorem is then re-derived from first principles for this particular situation. The previous problems are thereby eliminated, with the intention then being to proceed by taking the non-relativistic limit. In preparation for this step, separate expressions are obtained for the energy-momentum tensors of the field, particle and interaction, with the overall divergence being shown to be zero as required.

Chapter 8 takes the non-relativistic limit of the formulation in the previous chapter. Particular attention is paid to the appropriate expression for the energy-momentum tensor of the particle, so that certain subtleties can be addressed concerning rest energy and the symmetry of the tensors. Three rules are thereby identified which allow the non-relativistic limits for the field and interaction expressions to be obtained easily. The overall divergence is then confirmed to be zero for the non-relativistic case, showing that energy and momentum conservation have been successfully introduced into Bohm's model.

Finally, chapter 9 summarizes all the steps that have been taken in developing the argument and the problems encountered, including some comments on the strengths and weaknesses of the formulation.