

Energy and Momentum Conservation in Bohm's Model for Quantum Mechanics

Bryan Hall

Ph.D. Thesis

University of Western Sydney

2004

Abstract

Bohm's model for quantum mechanics is examined and a well-known drawback of the model is considered, namely the fact that the model does not conserve energy and momentum. It is shown that the Lagrangian formalism and the use of energy-momentum tensors provide a way of addressing this non-conservation aspect once the model is considered from the point of view of an interacting particle-field system. The full mathematical formulation that is then presented demonstrates that conservation can be reintroduced without disrupting the present agreement of Bohm's model with experiment.

Acknowledgement

I would most sincerely like to thank Dr Rod Sutherland for his continual guidance and suggestions, as well as his careful checking of the text of this thesis.

I would also like to thank the examiners for their work in preparing reports and suggesting various minor amendments to the thesis. The examiners made the following favourable comments:

Dr Sheldon Goldstein:

"This thesis is a very nice piece of work. It is written with exceptional clarity and facility of expression, and marks a significant contribution to the field."

Dr David Miller:

"The problem with Bohmian mechanics addressed by the thesis concerns energy and momentum conservation and is therefore fundamental. It is timely and important that the problem be addressed. The work in the thesis provides a solution to the problem and therefore makes a significant and original contribution to the discipline."

Dr Craig Callender:

"I recommend that the degree be awarded. Any errors I found were unimportant ones. My overall impression is that this dissertation is a very fine work in foundations of physics."

This is to certify that the work embodied in this thesis is original and has not been submitted for a higher degree at any other institution.

Bryan Hall

Contents

Chapter 1: Introduction	1
Chapter 2: Interpretations of Quantum Mechanics & the Measurement Problem ...	4
2.1 Historical Context.....	4
2.2 Mathematical Structure and Statistical Interpretation	5
2.3 The Correspondence Principle.....	6
2.4 The Copenhagen Interpretation	6
2.5 Hidden Variable Theories.....	9
Chapter 3: Bohm's Model	12
3.1 Summary of Bohm's Model.....	12
3.1.1 Equation of Continuity	13
3.1.2 Hamiltonian - Energy Considerations	15
3.1.3 Potential Gradient and Force in Bohm's Model.....	16
3.2 Velocity as a Function of Position - Bohm's Equation of Motion	17
3.3 Bohm's Model and Conventional Quantum Mechanics.....	19
3.4 Energy and Momentum Not Conserved	20
3.4.1 Restoring Conservation.....	22
3.5 Extensions to Bohm's Model	23
3.5.1 Holland's Generalisation	24
3.5.2 Deotto and Ghirardi's Generalisation	25
3.5.3 Sutherland's Generalisation.....	26
Chapter 4: Lagrangian Formalism	28
4.1 Lagrangian Formalism for Particle Motion.....	28
4.2 Lagrangian Formalism for Fields	30

4.3 Noether's Theorem & Conservation.....	33
4.4 Overall Lagrangian for a Particle & Field in Interaction	33
4.5 Squires attempted Lagrangian Formulation of Bohmian Mechanics	35
Chapter 5: A Lagrangian Formulation of Bohm's Model.....	38
5.1 Proposed Lagrangian Density.....	38
5.2 Derivation of Bohm's Equation of Motion from the Lagrangian Density	41
5.3 Field Equation Deriving from the Proposed Lagrangian Density	42
5.4 Consistency of the Derived Field Equation with Experiment.....	47
Chapter 6: Energy-Momentum Tensors.....	49
6.1 Basic Theory.....	49
6.2 Energy-Momentum Tensor for a Scalar Field.....	50
6.3 Energy and Momentum for a Scalar Field interacting with a Particle	51
6.3.1 Energy and Momentum Conservation Equations.....	51
6.3.2 Introduction of $T^{\mu\nu}_{\text{particle}}$	53
6.3.3 Global Equations.....	54
6.4 Tentative Application to Bohm's Model.....	55
Chapter 7: Relativistic Treatment.....	60
7.1 De Broglie's Model	61
7.2 Lagrangian Density for de Broglie's Model.....	63
7.3 Equation of Motion for the Particle.....	66
7.4 Field Equation	66
7.5 Energy-Momentum Tensor for the Particle.....	68
7.6 Noether's Theorem adapted to the Present Case.....	69
7.7 Summary of Equations describing Overall Conservation	72
7.8 Energy-Momentum Tensors $T^{\mu\nu}_{\text{field}}$ and $T^{\mu\nu}_{\text{interaction}}$	73

7.9 Divergence and Conservation.....	75
7.9.1 Divergence of $T^{\mu\nu}_{\text{field}}$	75
7.9.2 Divergence of $T^{\mu\nu}_{\text{particle}}$	76
7.9.3 Divergence of $T^{\mu\nu}_{\text{interaction}}$	76
7.9.4 Divergence of $T^{\mu\nu}_{\text{total}}$	77
Chapter 8: Non-Relativistic Limit	78
8.1 Non-Relativistic Energy-Momentum Tensor for the Particle	78
8.1.1 Physical Interpretation of $T^{\mu\nu}_{\text{particle}}$	78
8.1.2 Rules for obtaining the Non-Relativistic Limit.....	83
8.1.3 Derivation of $T^{\mu\nu}_{\text{particle}}$	85
8.2 Non-Relativistic Energy-Momentum Tensor for the Field	88
8.2.1 Non-Relativistic T^{ij}_{field}	91
8.2.2 Non-Relativistic T^{i0}_{field}	91
8.2.3 Non-Relativistic T^{0i}_{field}	91
8.2.4 Non-Relativistic T^{00}_{field}	92
8.2.5 Overall Non-Relativistic Result for $T^{\mu\nu}_{\text{field}}$	92
8.3 Non-Relativistic Energy-Momentum Tensor – Interaction Component.....	93
8.3.1 Non-Relativistic $T^{ij}_{\text{interaction}}$	95
8.3.2 Non-Relativistic $T^{i0}_{\text{interaction}}$	96
8.3.3 Non-Relativistic $T^{0i}_{\text{interaction}}$	96
8.3.4 Non-Relativistic $T^{00}_{\text{interaction}}$	96
8.3.5 Overall Non-Relativistic Result for $T^{\mu\nu}_{\text{interaction}}$	97
8.4 Divergence and Conservation.....	97
8.4.1 Divergence of $T^{\mu\nu}_{\text{field}}$	98
8.4.2 Divergence of $T^{\mu\nu}_{\text{particle}}$	99

8.4.3 Divergence of $T^{\mu\nu}_{\text{interaction}}$	101
8.4.4 Divergence of $T^{\mu\nu}_{\text{total}}$	103
8.5 Simplifications in the Bohmian Case	104
Chapter 9: Discussion and Conclusions	107
Appendix 1: Non Locality	109
A1.1 The EPR Paradox.....	109
A1.2 Bells Theorem.....	111
A1.3 Counterfactual Definiteness.....	112
A1.4 Bohm's Model and Non-locality.....	113
A1.5 Kochen and Specker's Proof	113
Appendix 2: Velocity Expression corresponding to the Modified Schrodinger Equation	115
Appendix 3: Rate of Change of a Particle's Energy in a Scalar Field	117
Appendix 4: Schrodinger Energy-Momentum Tensor	118
Appendix 5: Conservation Difficulty with the Schrodinger Energy-Momentum Tensor	120
Appendix 6: Viability of a Scalar Potential Description with de Broglie's Relativistic Model	122
Appendix 7: Relativistic Equation of Motion	123
A7.1 Derivation from the Relativistic Lagrangian Density.....	123
A7.2 Consistency of the Equation of Motion with the Identity $u_{\mu}u^{\mu} = c^2$	125
Appendix 8: Modified Klein-Gordon Equation	127
Bibliography	130

Please Note:

The symbol for the quantity “h bar” (= Planck’s constant divided by 2π) appears as **h** in the printing of this thesis (i.e., as a letter h with a small gap in its vertical stroke).