Appendix 6: Viability of a Scalar Potential Description with de Broglie's Relativistic Model

(An appendix to Chapter 7, Section 7.1)

In this appendix it will be shown that the basic definition [7-3]:

$$p^\mu = -\,\partial^\mu S \qquad \text{[A6-1]}$$

of de Broglie's model is compatible with the idea of motion under a scalar potential once we impose the condition that the particle has a variable mass given by [7-7]:

$$M = \frac{1}{c}\sqrt{(\partial_\mu S)\,(\partial^\mu S)} \qquad \text{[A6-2]}$$

Taking the τ derivative of equation [A6-1], we obtain:

$$\begin{aligned}
\frac{dp_\mu}{d\tau} &= -\,\frac{d(\partial_\mu S)}{d\tau} \\
&= -\,\frac{dx^\nu}{d\tau}\,\frac{\partial(\partial_\mu S)}{\partial x^\nu} \\
&= -\,u^\nu\,\partial_\nu\partial_\mu S \\
&= -\,\frac{p^\nu}{M}\,\partial_\mu\partial_\nu S \\
&= \frac{1}{M}\,(\partial^\nu S)\partial_\mu(\partial_\nu S)
\end{aligned}$$

Hence, inserting the definition [A6-2] for M, we have:

$$\begin{aligned}
\frac{dp_\mu}{d\tau} &= \frac{c}{\sqrt{(\partial_\lambda S)\,(\partial^\lambda S)}}\,(\partial^\nu S)\partial_\mu(\partial_\nu S) \\
&= \partial_\mu\left[\,c\sqrt{(\partial_\nu S)\,(\partial^\nu S)}\,\right]
\end{aligned}$$

and using the definition [7-4] for the quantum potential:

$$Q = c\sqrt{(\partial_\mu S)\,(\partial^\mu S)} - mc^2$$

we then arrive at the following equation:

$$\frac{dp_\mu}{d\tau} = \partial_\mu Q$$

In other words, the particle's equation of motion is then the relativistic version of "rate of change of momentum equals gradient of scalar potential", as required.