Appendix 1: Non Locality

(An appendix to Chapter 3, Section 3.3)

Since it is known that any hidden variable interpretation must incorporate "non-local" behaviour, we will look at the non-locality question in some detail and then examine briefly how Bohm's model deals with it.

A1.1 The EPR Paradox

In 1935, Einstein, Podolsky and Rosen published an objection to the Copenhagen interpretation of quantum mechanics[1] in an article which has come to be known as the "EPR paper". This paper demonstrated that the completeness of quantum mechanics, as interpreted by the reigning Copenhagen interpretation, could not be reconciled with the assumption of locality. The assumption of locality requires that, for any two particles, the result obtained by performing a measurement on one particle is independent of the type of measurement (if any) performed on the other particle when the two measurements events are separated by a space-like interval in space-time. By producing this demonstration, Einstein and his supporters presented a clear choice between locality and the assumption that quantum mechanics provided a "complete" or "sufficient" description of individual quantum entities.

It was the intention of the original EPR program to take locality as given and show that quantum mechanics could not be a complete theory in terms of describing individual quantum entities. This was achieved by introducing two essential definitions and then considering the case of two spatially separated quantum entities having correlated states. The definitions presented were as follows:

---

[1] Einstein A., Podolsky B. & Rosen N., *Can a Quantum Mechanical Description of Reality be Considered Complete?* Physics Review. Vol. 47, p. 777 (1935).

Definition 1: A necessary condition for a complete theory is that *"every element of physical reality must have a counterpart in the physical theory".*

Definition 2: A *sufficient* condition for identifying an element of reality is, *"If, without in anyway disturbing a system, we can predict with certainty (i.e., with probability equal to unity) the value of a physical quantity, then there must exist an element of reality corresponding to this physical quantity."*

Despite Toulmins comments (reported in Chapter 2) and Born's later assertion that *"the concept of reality is too much connected with emotions to allow a generally acceptable definition[2]",* <u>within the context of the definitions used in the EPR paper</u>, the EPR argument remains valid. In presenting EPR, it was intended that a resolution to the difficulties established would be obtained by admitting the existence of additional quantities consistent with quantum mechanics but restoring locality. The proposed introduction of such hidden values was clearly substantially at variance with the Copenhagen program. Ballentine[3] has concisely summarised Einstein's conclusions, and the contribution of the EPR Paper in general, as follows:

"The following two statements are incompatible:

(1) *The state vector provides a complete and exhaustive description of an individual system;*

(2) *The real physical conditions of spatially separated (non-interacting) objects are independent.*

Of course, one is logically free to accept either one of these statements (or neither). Einstein clearly accepted the second while Bohr apparently favoured the first. The importance of the EPR argument is that it proved for the first time that assuming the

[2] Born M., *Natural Philosophy of Cause and Chance.* Oxford University Press. London (1951).

[3] Ballentine L.E., *The Statistical Interpretation of Quantum Mechanics,* Reviews of Modern Physics Vol. 42, p. 363 (1970).

first statement above demands rejection of the second, and vice-versa, a fact that was not at all obvious before 1935, and which may not be universally realised today.”

A1.2 Bells Theorem

In the sometimes unsatisfactory debate that followed the presentation of the 1935 EPR paper, a very significant mile-stone was the later appearance of "Bell's theorem." Starting from the EPR argument, Bell went further and demonstrated that no hidden variable theory which leads to the predictions given by the quantum mechanical algorithm can be compatible with locality[4]. Bell achieved this by producing an inequality which constrains the distribution of measurement results possible for coincident events in EPR experiments assuming local hidden variables[5]. In a recent paper, Cramer[6] explains that Bell's inequality deals with the way in which the coincidence rate $\mathbf{R(\theta)}$ of an EPR experiment changes as θ starts from zero and becomes progressively larger. Bell proved mathematically that for all local hidden-variable theories the rate $\mathbf{R(\theta)}$ of coincident events in EPR type experiments must decrease linearly (or less rapidly) as θ increases, i.e., the fastest possible decrease in $\mathbf{R(\theta)}$ is proportional to θ. On the other hand quantum mechanics predicts that the coincidence rate is proportional to $\cos^2\theta$, so that for small θ it will decrease roughly as θ^2 (since $\cos^2\theta$ is approximately equal to $1-\theta^2$ for small θ). Therefore, quantum mechanics and locality require quantitatively different predictions about EPR measurements.

---

[4] In fact, Bell’s argument has been realised to be even more general than this, as will be discussed below.
[5] Bell J.S., *Physics* Vol.1, p.195 (1964).
[6] Cramer J., *Quantum Nonlocality and the Possibility of Superluminal Effects.* Published in the Proceedings of the NASA Breakthrough Propulsion Physics Workshop, Cleveland, OH, (August 12-14, 1997). (Also available from http://www.npl.washington.edu/npl/int_rep/qm_nl.html).

In response to Bells Theorem, experiments using spatially separated but correlated quantum entities (mainly pairs of photons) have been undertaken[7]. These experiments have verified the standard quantum mechanical predictions and, in so doing, have ruled out local hidden variable theories. It is generally agreed that the non-local quantum correlations observed are not in direct contradiction with relativity since they do not permit information transfer between space-like separated points (in particular, they do not permit faster-than-light signalling).

Unfortunately, the first experimental results from EPR experiments were frequently incorrectly interpreted as demonstrating the inadmissibility of hidden variable theories. However, since Bell's theorem assumes only a local hidden variable theory, the possibility of non-local hidden variable theories satisfying both Bell's theorem and the experimentally verifiable predictions of quantum mechanics remains open. In other words, from the viewpoint of hidden variables advocates, Bells theorem and the subsequent experiments mentioned above have simply demonstrated that locality, not hidden variable models, conflicts with experiment. Bohm’s theory is an example of a non-local hidden variable theory that is consistent with the experimentally testable predictions of quantum mechanics.

A1.3 Counterfactual Definiteness

Later it became clear that the argument was, in fact, even more general and quantum mechanics cannot even be compatible with both locality and Stapps' assumption of counterfactual definiteness, where the latter is defined as follows:

[7] Freedman and Clauser demonstrated a 6σ violation of Bell's inequality in 1972. Freedman S.J. & Clauser J.F., Physical Review Letters, Vol. 28, pp. 938-941 (1972).
Aspect et al demonstrated a 46σ violation of Bell's inequality in 1982. Aspect A , Dalibard J. & Roger G., Physical Review Letters Vol. 49, pp. 91 & 1804 (1982).

"For each particle on which a measurement is performed, a definite value would have been found if a different spin component had been measured on it instead (although we cannot know what the specific value would have been) and, furthermore, the complete set of such values (measured and unmeasured together) can be meaningfully discussed".[8]

A1.4 Bohm's Model and Non-locality

Bohm's model deals with the required non-locality as follows: Since a correlated pair of particles is described by a single, overall wavefunction, a measurement on one of the particles must have an effect on the wavefunction description of the other particle. Then, by the nature of Bohm's model, this also necessarily (and instantaneously) affects the second particle's hidden position and momentum. Thus Bohm's theory automatically incorporates an explicit description of the non-locality implied by Bell's theorem. It does this, however, at the expense of a conflict with the principle of relativity, albeit a hidden one. Hardy[9] has argued that such a hidden conflict with the equality of all reference frames may be a necessary feature of any hidden variable model for quantum mechanics.[10]

A1.5 Kochen and Specker's Proof

Kochen and Specker presented a proof[11] which showed that any hidden variable theory must also be "contextual," viz, the value obtained by a measurement must sometimes depend on what other observable happens to be measured at the same time (i.e., the value obtained depends on the "context"). In other words, the observable

[8] Stapp H., p. 637-652 in *Symposium on the Foundations of Modern Physics*, Edited by Lahti P. & Mittelstaedt P. World Scientific Publishing Co. (1985).
[9] Hardy L., Physical Review Letters Vol. 68, p. 2981 (1992); Hardy L. & Squires E.J., Physics Letters Vol. A168, p. 169 (1992).
[10] However, the hyperspace Bohm-Dirac model in Durr D., Goldstein S., Munch-Berndl K., et al, Physical Review A, Vol. 60, pp. 2729-2736 (1999), can be considered a counterexample to this claim.

values can't all just be pre-existing and waiting to be measured. If observables do have values before they are measured, then measurements do not in general yield those values. This is indeed the case for Bohm's model, since the measurement outcomes obtained don't exist prior to the measurement. Rather, the measured values are created during the measurement process, i.e., during the gradual spatial separation of the wave function into non-overlapping wave packets. In the case of momentum, for example, the measured value replaces the pre-existing value during this time, whilst for other observables (such as spin in Bell's extension of Bohm's model[12]) there may be no pre-existing value at all beforehand.

The Kochen and Specker proof (and those of other people, such as Gleason[13]) was rather complicated and Mermin[14] has pointed out that Bell's theorem essentially proves the same thing more simply (in addition to its implications about non-locality).

---

[11] Kochen S. and Specker E.P., Journal of Mathematics and Mechanics Vol. 17, p. 59 (1967).
[12] Bell J.S., Paper 4 in *Speakable and Unspeakable in Quantum Mechanics*. Cambridge University Press (1987).
[13] Gleason A.M., Journal of Mathematics and Mechanics Vol. 6, p. 885 (1957).
[14] Mermin N.D., Physical Review Letters Vol. 65, pp. 3373-3376 (1990); Reviews of Modern Physics Vol. 65, pp. 803-815 (1993).